Math 72 5.5 Special Products

Objectives

1) Factor special patterns
   - difference of two squares
   - perfect square trinomials
   - difference of cubes
   - sum of cubes.

2) Recognize factors which are prime and cannot be factored
   - sum of squares
   - sum of a square and a non-square
   - difference of a square and a non-square
   - trinomial from a difference of cubes (if degree 2)
   - trinomial from a sum of cubes (if degree 2)

Multiply

① $(4x-3)(4x+3)$

$= 16x^2 + 12x - 12x - 9$

$= \boxed{16x^2 - 9}$

This pattern is called a <u>difference of two squares</u>

subtract — perfect squares

② $(4x-3)(4x-3)$

$= 16x^2 - 12x - 12x + 9$

$= \boxed{16x^2 - 24x + 9}$

③ $(4x+3)(4x+3)$

$= 16x^2 + 12x + 12x + 9$

$= \boxed{16x^2 + 24x + 9}$

These two are the pattern called a <u>perfect square trinomial</u>.

④ $(4x+3)(16x^2 - 12x + 9)$

$= 64x^3 - \cancel{48x^2} + \cancel{36x}$

$\quad + \cancel{48x^2} - \cancel{36x} + 27$

$= \boxed{64x^3 + 27}$

This pattern is called the <u>sum of two cubes</u>

add — perfect cubes

⑤ $(4x-3)(16x^2 + 12x + 9)$

$= 64x^3 + \cancel{48x^2} + \cancel{36x}$

$\quad - \cancel{48x^2} - \cancel{36x} - 27$

$= \boxed{64x^3 - 27}$

This pattern is called the <u>difference of two cubes</u>

subtract — perfect cubes

⑥ $(4x-3)^3$

$= (4x-3)(4x-3)(4x-3)$

$= (4x-3)(16x^2 - 24x + 9)$

$= 64x^3 - 96x^2 + 36x$

$\quad - 48x^2 + 72x - 27$

$= \boxed{64x^3 - 144x^2 + 108x - 27}$

This pattern is called a <u>perfect cube</u>.

We will not factor this! But it is a common error to confuse it with sum or difference of cubes.

Factor

⑦ $16x^2 - 25$

$= \boxed{(4x-5)(4x+5)}$

Difference of Squares

notice

1) two terms
2) subtracted
3) $16 = 4^2$ is a perfect square
   $x^2 = x \cdot x$ "
   $25 = 5^2$ "

Use $\sqrt{25} = 5$ and $\sqrt{16} = 4$ with one added factor and one subtracted.

$$a^2 - b^2 = (a-b)(a+b)$$

⑧ $x^2 + 2y^2$

$= \boxed{\text{prime}}$

1) two terms
2) added ☹

⑨ $4x^6 - y^4$

$= \boxed{(2x^3 - y^2)(2x^3 + y^2)}$

1) two terms
2) subtracted
3) $\left.\begin{array}{l} 4 = 2 \cdot 2 \\ x^6 = x^3 \cdot x^3 \\ y^4 = y^2 \cdot y^2 \end{array}\right\}$ perfect squares.

⑩ $6a^3 - 24a$

$= 6a(a^2 - 4)$

$= \boxed{6a(a-2)(a+2)}$

GCF!

⑪ $(n+1)^2 - 9$

$= [(n+1) - 3][(n+1) + 3]$

$= [n+1-3][n+1+3]$

$= \boxed{(n-2)(n+4)}$

1) 2 terms
2) difference
3) squares $(n+1)^2$
   $9 = 3^2$

(12) $16x^4-1$

$= (4x^2-1)(4x^2+1)$

$= \boxed{(2x-1)(2x+1)(4x^2+1)}$

$\left.\begin{array}{l} 16=4^2 \\ x^4=x^2\cdot x^2 \\ 1=1^2 \\ 4x^2=(2x)^2 \end{array}\right\}$ diff of sq.

(13) $12r^2-3t^4$

$= 3(4r^2-t^4)$

$= \boxed{3(2r-t^2)(2r+t^2)}$

GCF!

$\left.\begin{array}{l} 4=2^2 \\ r^2 \\ t^4=r^2\cdot r^2 \end{array}\right\}$ diff of sq.

(14) $\underbrace{x^3+3x^2}\underbrace{-4x-12}$

$= x^2(x+3)-4(x+3)$

$= (x+3)(x^2-4)$

diff of sq

$= \boxed{(x+3)(x-2)(x+2)}$

4 terms!
grouping

(15) $x^2+10x+25$

$= \boxed{(x+5)(x+5)}$

check by FOIL

$= x^2+5x+5x+25$

$= x^2+10x+25$

(16) $x^2+11x+25$

$(x+5)(x+5)$

$= x^2+10x+25$ NO.

25

11

1, 25
5, 5
No

$\boxed{\text{PRIME}}$

Perfect Square Trinomial

Notice: 1) 3 terms
2) last term is added
3) 1st and last term are perfect squares
x^2
$25=5^2$

Use these square roots $\sqrt{1}=1$ $\sqrt{25}=x$
with the sign on the middle term
$+10x \Rightarrow$ use $+$

* CAUTION: Always check by FOIL. Some appear to be perfect squares when they aren't.

$a^2+2ab+b^2 = (a+b)(a+b) = (a+b)^2$

$a^2-2ab+b^2 = (a-b)(a-b) = (a-b)^2$

(17) $81x^2 - 72x + 16$

$= \boxed{(9x-4)(9x-4)}$

$= 81x^2 - 36x - 36x + 16$

$= 81x^2 - 72x + 16$ ✓

3 terms
last term added
$\left.\begin{array}{l} 81x^2 \rightarrow (9x)^2 \\ 16 \rightarrow (4)^2 \end{array}\right\}$ perfect squares

(18) $x^3 - 8$

$= \boxed{(x-2)(x^2 + 2x + 4)}$

same sign; $x \cdot x$; opposite sign; $x \cdot 2$; always positive; $2 \cdot 2$

check by multiplying

$= x^3 + \cancel{2x^2} + \cancel{4x}$
$\quad - \cancel{2x^2} - \cancel{4x} - 8$

$= x^3 - 8$ ✓

Difference of 2 cubes

Notice: 1) 2 terms
2) subtracted
3) perfect cubes
$(x)^3$
$8 = (2)^3$

Use cube roots $\sqrt[3]{x^3} = x \rightarrow a$
$\sqrt[3]{8} = 2 \rightarrow b$

$a^3 - b^3 = (\underbrace{a-b}_{\text{degree 1 terms}})(\underbrace{a^2 + ab + b^2}_{\text{degree 2 terms}})$

−	−	+	+
original sign	same S	opposite O	always positive AP

"SOAP"

(19) $x^3 + 8$

$= \boxed{(x+2)(x^2 - 2x + 4)}$

check by multiplying

$= x^3 - \cancel{2x^2} + \cancel{4x}$
$\quad + \cancel{2x^2} - \cancel{4x} + 8$

$= x^3 + 8$ ✓

Sum of 2 cubes

1) 2 terms
2) added
3) perfect cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
S O AP

If the trinomial is degree 2 it's prime.

20. $27x^3 - 64y^3$

$= \boxed{(3x-4y)(9x^2+12xy+16y^2)}$

$\sqrt[3]{27} = 3$

$\sqrt[3]{64} = 4$

$\sqrt[3]{x^3} = x$

$\sqrt[3]{y^3} = y$

$a = 3x$

$b = 4y$

$(a-b)(a^2+ab+b^2)$

$(3x)^2 = 9x^2$

$(3x)(4y) = 12xy$

$(4y)^2 = 16y^2$

21. $125x^3y^6 + 1$

$= \boxed{(5xy^2+1)(25x^2y^4 - 5xy^2 + 1)}$

$\sqrt[3]{125} = 5$

$\sqrt[3]{x^3} = x$

$\sqrt[3]{y^6} = y^2$

$\sqrt[3]{1} = 1$

$a = 5xy^2$

$b = 1$

$(a+b)(a^2-ab+b^2)$

$(5xy^2)^2 = 25x^2y^4$

$5xy^2 \cdot 1$

$= 5xy^2$

$1^2 = 1$